Densities, Parton Distributions, and Measuring the Non-Spherical Shape of the Nucleon

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Spin-dependent quark densities, matrix elements of specific density operators in proton states of definite spin-polarization, indicate that the nucleon may harbor an infinite variety of non-spherical shapes. We show that these matrix elements are closely related to specific transverse momentum dependent parton distributions accessible in the angular dependence of the semi-inclusive processes $ep \to e\pi X$ and the Drell-Yan reaction $pp \to l\bar{l}X$. New measurements or analyses would allow the direct exhibition of the non-spherical nature of the proton.

Since the discovery that that the spins of quarks and anti-quarks account for only about 33% of the nucleon spin [1], [2], many experiments have sought the origins of the remainder, which must be accounted for by effects of quark and gluon angular momentum. The importance of orbital angular momentum is also demonstrated in exclusive reactions. Measurements [3, 4] find that the ratio of the proton's electric and magnetic form factor G_E/G_M , falls with increasing momentum transfer Q^2 for $1 < Q^2 < 6$ GeV². This striking behavior indicates that the sum of the orbital angular momentum of the quarks in the proton is non-vanishing [5, 6, 7, 8].

One expects that the presence of significant orbital angular momentum would lead to a non-spherical shape, if such can be defined by an appropriate operator. We showed [9], using the proton model of Ref. [10], that the rest-frame ground-state matrix elements of spin-dependent density operators reveal a host of non-spherical shapes. The use of the spin-dependent density operator allows the detailed connection between orbital, spin and total angular momentum to be revealed in quantum systems. In the model of [10] the orbital angular momentum originates from the relativistic nature of the quarks manifest by lower components of Dirac spinors of the wave function, but there are many other potential sources.

It is natural to ask if the non-spherical nucleonic shapes can be measured. While matrix elements of the non-relativistic spin-density operator have been measured in condensed matter systems [11] to reveal highly non-spherical densities and the related the orbital angular momentum content of electron orbitals, finding the corresponding determination of the nucleon properties has remained a challenge. Our purpose here is to show that matrix elements of the spin-dependent density are closely related to specific unintegrated (transverse momentum dependent) parton densities that could be obtained by measuring the angular dependence of the $ep \to e\pi X$ reaction and of the Drell-Yan production cross section in pp collisions.

We begin by explaining how the shapes of a nucleon are exhibited by studying the restframe ground-state matrix elements of spin-dependent density operators [9]. The usual density operator in non-relativistic quantum mechanics is given by

$$\widehat{\rho}(\mathbf{r}) = \sum_{i} \delta(\mathbf{r} - \mathbf{r}_{i}), \tag{1}$$

where \mathbf{r}_i is the position operator of the *i*'th particle. Matrix elements of this operator yield the density of a system. Suppose the particles also have spin 1/2. Then one can measure the probability that particle is at a given position \mathbf{r} and has a spin in an arbitrary, fixed direction specified by a unit vector \mathbf{n} . The spin projection operator is $(1 + \boldsymbol{\sigma} \cdot \mathbf{n})/2$, so the spin-dependent density operator is

$$\widehat{\rho}(\mathbf{r}, \mathbf{n}) = \sum_{i} \delta(\mathbf{r} - \mathbf{r}_{i}) \frac{1}{2} (1 + \boldsymbol{\sigma}_{i} \cdot \mathbf{n}).$$
(2)

The spin-dependent density allows the presence of the orbital angular momentum to be revealed in the shape of the computed density. To understand this, it is worthwhile to consider a simple example of a single charged particle moving in a fixed rotationally invariant potential in an energy eigenstate $|\Psi\rangle$ of quantum numbers: l=1, j=1/2, polarized in the direction $\hat{\mathbf{s}}$ and radial wave function R(r). We find

$$\rho(\mathbf{r}, \mathbf{n}) = \langle \Psi | \hat{\rho}(\mathbf{r}, \mathbf{n}) | \Psi \rangle = \frac{R^2(r)}{2} \langle \hat{\mathbf{s}} | 1 + 2\boldsymbol{\sigma} \cdot \hat{\mathbf{r}} \, \mathbf{n} \cdot \hat{\mathbf{r}} - \boldsymbol{\sigma} \cdot \mathbf{n} | \hat{\mathbf{s}} \rangle.$$
(3)

Suppose $\hat{\mathbf{n}}$ is either parallel or anti-parallel to the direction of the proton angular momentum defined by the vector $\hat{\mathbf{s}}$. The direction of the vector $\hat{\mathbf{s}}$ defines an axis (the "z-axis"), and the direction of vectors can be represented in terms of this axis: $\hat{\mathbf{s}} \cdot \hat{\mathbf{r}} = \cos \theta$. With this notation $\rho(\mathbf{r}, \mathbf{n} = \hat{\mathbf{s}}) = R^2(r)\cos^2\theta$, $\rho(\mathbf{r}, \mathbf{n} = -\hat{\mathbf{s}}) = R^2(r)\sin^2\theta$ and the non-spherical shape is exhibited. The average of these two cases is a spherical shape, as is the average over the direction of $\hat{\mathbf{s}}$ or the average over the direction of \mathbf{n} .

The densities of Eq. (1) and Eq. (2) can be extended to include other operators. Indeed, Ref. [11] uses only the spin-dependent term appearing in Eq. (2), and this is weighted by the electronic charge. For systems of quarks, the densities could be weighted by the charge of the quarks, or be concerned with a specific flavor. One could also weight the spin-dependence by other operators. In particular, consider

$$\widehat{\rho}_{REL}(\mathbf{r}, \mathbf{n}) \equiv \sum_{i} \delta(\mathbf{r} - \mathbf{r}_i) \frac{1}{2} (1 + \gamma_i^0 \boldsymbol{\sigma}_i \cdot \mathbf{n}), \tag{4}$$

where the relativistic aspects are emphasized by the appearance of the Dirac operator γ^0 , which becomes unity in the non-relativistic limit. We denote the density operators of Eq. (1),Eq. (2) and Eq. (4), and any number of obvious extensions, simply as densities.

These densities are defined in terms of position, but to use QCD it is necessary to define operators that give the probability for a particle to have a given momentum, \mathbf{K} , and a given direction of spin, \mathbf{n} . The field-theoretic version of the spin-dependent charge density operator, Eq. (2), is a generalization of the operator defined in Ref. [9]:

$$\widehat{\rho}(\mathbf{K}, \mathbf{n}) = \int \frac{d^3 \xi}{(2\pi)^3} e^{-i\mathbf{K}\cdot\boldsymbol{\xi}} \, \overline{\psi}(0) (\gamma^0 + \boldsymbol{\gamma} \cdot \mathbf{n}\gamma_5) \mathcal{L}(0, \xi; \, \text{path}) \psi(\boldsymbol{\xi}) \Big|_{t=\xi^0=0},$$
 (5)

where ψ is a quark field operator and flavor indices are omitted. The quark field operators are evaluated at equal time and accompanied by a path-ordered exponential link operator $\mathcal{L}((0,\xi;\,\text{path}))$ needed for color-gauge invariance. This introduces a path-dependence, which

must be specified correctly to obtain a parton interpretation. We will use the choice of Ref. [12]; see below. The first quantized version of Eq. (5) (neglecting the gluonic aspects) is of the form of Eq. (2) except that the factor $\delta(\mathbf{r} - \mathbf{r}_i)$ is replaced by $\delta(\mathbf{K} - \mathbf{K}_i)$, where \mathbf{K}_i is the momentum of the i'th quark. It is worthwhile to define another density corresponding to $\widehat{\rho}_{\text{REL}}$ of Eq. (4). This is given by

$$\widehat{\rho}_{\text{REL}}(\mathbf{K}, \mathbf{n}) = \int \frac{d^3 \xi}{(2\pi)^3} e^{-i\mathbf{K}\cdot\boldsymbol{\xi}} \left. \bar{\psi}(0) \gamma^0 (1 + \boldsymbol{\gamma} \cdot \mathbf{n} \gamma_5) \mathcal{L}(0, \boldsymbol{\xi}; \text{ path}) \psi(\boldsymbol{\xi}) \right|_{t=\xi^0=0}.$$
 (6)

The matrix element of a density operator in a nucleon state $|P,S\rangle$ of definite total angular momentum defined by four-vector S^{μ} and momentum P is

$$\rho(\mathbf{K}, \mathbf{n}, \mathbf{S}) \equiv \langle P, S | \hat{\rho}_{G}(\mathbf{K}, \mathbf{n}) | P, S \rangle, \ \rho_{REL}(\mathbf{K}, \mathbf{n}, \mathbf{S}) \equiv \langle P, S | \hat{\rho}_{REL}(\mathbf{K}, \mathbf{n}) | P, S \rangle.$$
 (7)

The most general shape of the proton in its *rest frame*, obtained if parity and rotational invariance are upheld is then [9]

$$\rho(\mathbf{K}, \mathbf{n}, \mathbf{S}) = A(\mathbf{K}^2) + B(\mathbf{K}^2)\mathbf{n} \cdot \hat{\mathbf{S}} + C(\mathbf{K}^2) \left(\mathbf{n} \cdot \hat{\mathbf{K}} \, \hat{\mathbf{S}} \cdot \hat{\mathbf{K}} - \frac{1}{3} \mathbf{n} \cdot \hat{\mathbf{S}} \right)$$

$$\rho_{\text{REL}}(\mathbf{K}, \mathbf{n}, \mathbf{S}) = A_{\text{REL}}(\mathbf{K}^2) + B_{\text{REL}}(\mathbf{K}^2)\mathbf{n} \cdot \hat{\mathbf{S}} + C_{\text{REL}}(\mathbf{K}^2) \left(\mathbf{n} \cdot \hat{\mathbf{K}} \, \hat{\mathbf{S}} \cdot \hat{\mathbf{K}} - \frac{1}{3} \mathbf{n} \cdot \hat{\mathbf{S}} \right), (8)$$

with the last terms generating the non-spherical shape. Any wave function that yields a non-zero value of the coefficient $C(\mathbf{K}^2)$ or $C_{\text{REL}}(\mathbf{K}^2)$ represents a system of a non-spherical shape. If the relativistic constituent quark model of [10] is used, the principle difference between $\rho(\mathbf{K}, \mathbf{n}, \mathbf{S})$ and $\rho_G(\mathbf{K}, \mathbf{n}, \mathbf{S})$ would be that $C_{\text{REL}} = -C$. This indicates that either C_{REL} or C can be used to infer information about the possible shapes of the nucleon. Measuring either $C(\mathbf{K}^2)$ or $C_{\text{REL}}(\mathbf{K}^2)$, would require controlling the three different vectors \mathbf{n}, \mathbf{S} and \mathbf{K} or their equivalent.

The densities of Eq. (8) are difficult to measure because the system must be probed without momentum transfer and the initial and final states are the same. This configuration also appears in parton distributions, both ordinary and transverse-momentum-dependent TMD. Parton density operators depend on pairs of quark-field operators defined at a fixed light cone time $\xi^+ = \xi^3 + \xi^0 = 0$ while the density operators of Eq. (5) and Eq. (6) are defined as an equal-time, $\xi^0 = 0$, correlation functions and cannot be regarded as parton density operators. However, we find a relation between the two sets of operators by integrating Eqs. (5,6) over all values of K_z . This sets $\xi^3 = 0$, so the quark field operators of Eq. (5) or Eq. (6) are now evaluated at $\xi^0 = 0$ and $\xi^3 = 0$ so $\xi^{\pm} = 0$:

$$\hat{\rho}_{T}(\mathbf{K}_{T}, \mathbf{n}) \equiv \int_{-\infty}^{\infty} dK_{z} \hat{\rho}(\mathbf{K}, \mathbf{n}) = \int \frac{d^{2}\xi_{T}}{(2\pi)^{2}} e^{-i\mathbf{K}_{T}\cdot\boldsymbol{\xi}_{T}} \left. \bar{\psi}(0)(\gamma^{0} + \boldsymbol{\gamma} \cdot \mathbf{n}\gamma_{5}) \mathcal{L}(0, \xi; \ n^{-}) \psi(\boldsymbol{\xi}_{T}) \right|_{\xi^{\pm}=0}$$

$$\hat{\rho}_{RELT}(\mathbf{K}_{T}, \mathbf{n}) \equiv \int_{-\infty}^{\infty} dK_{z} \hat{\rho}_{REL}(\mathbf{K}, \mathbf{n}) = \int \frac{d^{2}\xi_{T}}{(2\pi)^{2}} e^{-i\mathbf{K}_{T}\cdot\boldsymbol{\xi}_{T}} \left. \bar{\psi}(0) \gamma^{0} (1 + \boldsymbol{\gamma} \cdot \mathbf{n}\gamma_{5}) \mathcal{L}(0, \xi; \ n^{-}) \psi(\boldsymbol{\xi}_{T}) \right|_{\xi^{\pm}=0} (9)$$

where the specific path n^- is that of Appendix B of [12]. To obtain the relevant transverse densities we take the matrix element of $\hat{\rho}_{GT}$ in a nucleon state polarized in the transverse direction \mathbf{S}_T , with

$$\rho_{T}(\mathbf{K}_{T}, \mathbf{n}, \mathbf{S}_{T}) \equiv \langle P, \mathbf{S}_{T} | \hat{\rho}_{T}(\mathbf{K}_{T}, \mathbf{n}) | P, \mathbf{S}_{T} \rangle \tag{10}$$

$$= A_{T}(K_{T}^{2}) + B_{T}(K_{T}^{2}) \mathbf{n} \cdot \hat{\mathbf{S}}_{T} + C_{T}(K_{T}^{2}) \frac{(\mathbf{n} \cdot \mathbf{K}_{T} \hat{\mathbf{S}}_{T} \cdot \mathbf{K}_{T} - \frac{1}{2}K_{T}^{2} \mathbf{n} \cdot \hat{\mathbf{S}}_{T})}{M^{2}},$$

$$\rho_{RELT}(\mathbf{K}_{T}, \mathbf{n}, \mathbf{S}_{T}) = A_{RELT}(K_{T}^{2}) + B_{RELT}(K_{T}^{2}) \mathbf{n} \cdot \hat{\mathbf{S}}_{T} + C_{RELT}(K_{T}^{2}) \frac{(\mathbf{n} \cdot \mathbf{K}_{T} \hat{\mathbf{S}}_{T} \cdot \mathbf{K}_{T} - \frac{1}{2}K_{T}^{2} \mathbf{n} \cdot \hat{\mathbf{S}}_{T})}{M^{2}} \tag{11}$$

where M is the nucleon mass, and we take the unit vector \mathbf{n} to be in the transverse direction. Information regarding the shape of the nucleon resides in the functions C_T , $C_{\text{REL}T}$. We now are able to connect our newly-defined transverse densities with TMD parton distributions. The latter are related to Dirac projections of correlation functions [12]:

$$\Phi^{[\Gamma]}(x, \mathbf{K}_T) = \int \frac{d\xi^- d^2 \xi_T}{2 (2\pi)^3} e^{iK \cdot \xi} \langle P, S | \overline{\psi}(0) \Gamma \mathcal{L}(0, \xi; n_-) \psi(\xi) | P, S \rangle \bigg|_{\xi^+ = 0}.$$
 (12)

The projections $\Phi^{[\Gamma]}$ depend on the fractional momentum $x = K^+/P^+$, \mathbf{K}_T and on the hadron momentum P (in essence only P^+ and M where we work in a frame in which $P^+ \gg M$.). Depending on the Lorentz structure of the Dirac matrix Γ the projections $\Phi^{[\Gamma]}$ are ordered according to powers of M/P^+ multiplied with a function depending only on x and \mathbf{K}_T^2 . Each factor M/P^+ leads to a suppression by a power in cross sections [13] so that one may refer to the projections as having a 'twist' t related to the power $(M/P^+)^{t-2}$ that appears. Then moments (in x) of the \mathbf{K}_T -integrated functions involve local operators of twist t [14].

We use certain transverse momentum dependent parton distribution functions of [12], which for transversely polarized nucleons are given by

$$\Phi^{[\gamma^+]}(x, \mathbf{K}_T) = f_1(x, K_T^2), \tag{13}$$

$$\Phi^{[i\sigma^{i+}\gamma_5]}(x, \mathbf{K}_T) = S_T^i h_1(x, K_T^2) + \frac{\left(K_T^i K_T^j - \frac{1}{2} K_T^2 \delta_{ij}\right) S_T^j}{M^2} h_{1T}^{\perp}(x, K_T^2), \tag{14}$$

$$\Phi^{[\gamma^i \gamma_5]}(x, \mathbf{K}_T) = \frac{M S_T^i}{P^+} g_T(x, K_T^2) + \frac{M}{P^+} \frac{\left(K_T^i K_T^j - \frac{1}{2} K_T^2 \delta_{ij}\right) S_T^j}{M^2} g_T^{\perp}(x, K_T^2). \tag{15}$$

Terms of higher order in (M/P^+) are neglected in the extraction of the functions g_T, g_T^{\perp} from high energy data. Similarly, at high energies, we may replace γ^+ by $\sqrt{2}\gamma^0$. Note that the quantity g_T^{\perp} is closely related to the quantity C_T , while the quantity h_{1T}^{\perp} is closely related to C_{RELT} because $i\sigma^{i+}\gamma^5 = \gamma^+\gamma^i\gamma^5 \to \sqrt{2}\gamma^0\gamma^i\gamma^5$. Extracting g_T^{\perp} would require a higher twist analysis, while h_{1T}^{\perp} appears at leading order in the cross sections for semi-inclusive leptoproduction experiments [15]. Thus the relativistic spin-dependent density Eq. (11) is easier to measure than the quantity of Eq. (10).

We integrate the above parton distribution functions over all x so that the field operators are evaluated at $\xi^{\pm} = 0$ as in our spin-dependent densities. A tilde is placed over a given

quantity to define the x-integrated result, e.g. $\tilde{\Phi}^{[\Gamma]}(\mathbf{K}_T) \equiv \int dx \Phi^{[\Gamma]}(x, \mathbf{K}_T)$, $\tilde{f}_1(K_T^2) \equiv \int dx f_1(x, K_T^2)$, etc. Then

$$\widetilde{\Phi}^{[\Gamma]}(\mathbf{K}_T) = \int \frac{d^2 \xi_T}{2P^+ (2\pi)^2} e^{-i\mathbf{K}_T \cdot \boldsymbol{\xi}_T} \langle P, S | \overline{\psi}(0) \Gamma \mathcal{L}(0, \xi; n_-) \psi(\boldsymbol{\xi}_T) | P, S \rangle \bigg|_{\xi^+ = 0, \xi^- = 0}.$$
(16)

The various $\Phi^{[\Gamma]}$ are expressed, in the infinite momentum frame, in terms of transverse momentum dependent parton distribution functions Eq. (14) and Eq. (15). To relate these to our functions A_T, B_T, C_T we evaluate these equations in the rest frame $(P^+ \to M/\sqrt{2}, \gamma^+ \to \sqrt{2}\gamma^0)$ and choose the operators Γ to correspond to those appearing in the spin-dependent densities. We find that

$$\sqrt{2}\rho(\mathbf{K}_{T},\mathbf{n},\mathbf{S}_{T}) = \tilde{f}_{1}(K_{T}^{2}) + \tilde{g}_{T}(K_{T}^{2})\mathbf{n} \cdot \hat{\mathbf{S}}_{T} + \frac{\left(\hat{\mathbf{n}}_{T} \cdot \mathbf{K}_{T}\hat{\mathbf{S}}_{T} \cdot \mathbf{K}_{T} - \frac{1}{2}K_{T}^{2}\hat{\mathbf{n}} \cdot \hat{\mathbf{S}}_{T}\right)}{M^{2}} \tilde{g}_{T}^{\perp}(K_{T}^{2}), (17)$$

$$\sqrt{2}\rho_{\text{REL}T}(\mathbf{K}_{T},\mathbf{n},\mathbf{S}_{T}) = \tilde{f}_{1}(K_{T}^{2}) + \tilde{h}_{1}(K_{T}^{2})\mathbf{n} \cdot \hat{\mathbf{S}}_{T} + \frac{\left(\hat{\mathbf{n}}_{T} \cdot \mathbf{K}_{T}\hat{\mathbf{S}}_{T} \cdot \mathbf{K}_{T} - \frac{1}{2}K_{T}^{2}\hat{\mathbf{n}} \cdot \hat{\mathbf{S}}_{T}\right)}{M^{2}} \tilde{h}_{1T}^{\perp}(K_{T}^{2}). \tag{18}$$

Finding a non-zero value of either \tilde{g}_T or \tilde{h}_{1T}^{\perp} would demonstrate that the proton is not spherical. Both the spectator model [16] and the quark model [10] yield the result that $\tilde{g}_T = -\tilde{h}_{1T}^{\perp}$. This relation does not appear to be a general result. In particular, one may use the most general parameterization of the correlation functions [12, 17] in terms of scalar functions $A_i(K,P)$ to show that g_T^{\perp} is proportional to the term A_8 and h_{1T}^{\perp} is proportional to the term P^+/MA_{11} . This means that, in the nucleon rest frame, the ratio $g_T^{\perp}/h_{1T}^{\perp}$ is simply a function of (x, \mathbf{K}_T^2) . Furthermore, the quantity \tilde{h}_{1T}^{\perp} is known to characterize the dependence of the transverse polarization of quarks in a transversely polarized nucleon on the direction of \mathbf{K}_T [18, 19]. Thus the two quantities \tilde{g}_T^{\perp} and \tilde{h}_{1T}^{\perp} each characterize the non-spherical nature of the nucleon, and the density ρ_{RELT} can be thought of as "the" spin-dependent density".

Next we focus on experimental means to access \tilde{h}_{1T}^{\perp} . The presence of the term \tilde{h}_{1T}^{\perp} causes distinctive signatures in semi-inclusive leptoproduction experiments [15] in which a hadron h is produced. If the target is polarized in a direction transverse to the lepton scattering plane, the cross section acquires a term proportional to $\cos(3\phi_h^l)$ where ϕ_h^l is the angle between the hadron production plane (defined by the momenta of the incoming virtual photon and the outgoing hadron) and the lepton scattering plane. A similar effect occurs in electroweak semi-inclusive deep inelastic leptoproduction and this could be accessed at high energies such as those found at HERA [20]. Another signature occurs in the angular distribution of the leptoproduction of ρ mesons [21], obtained using an unpolarized lepton beam and a transversely polarized target. Similarly the term \tilde{h}_{1T}^{\perp} makes its presence felt in studying the production of two-pions inside the same current jet [22]. In each of these cases, the momentum of the virtual photon and its vector nature provide the analogue of two of the three vectors \mathbf{n} and \mathbf{S}_T needed to define the spin-dependent density. The hadronic transverse momentum provides the third, \mathbf{K}_T .

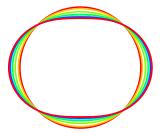


FIG. 1: (Color online) Transverse shapes of the nucleon: $\sqrt{2}\hat{\rho}_T(\mathbf{K}_T,\mathbf{n})/\tilde{f}_1(K_T^2)$. The horizontal axis is the the direction of \mathbf{S}_T and $\mathbf{n} = \hat{\mathbf{S}}_T$, $\phi_n = 0$. The shapes vary from circular to highly deformed as K_T is increased from 0 to 2.0 GeV in steps of 0.25 GeV.



FIG. 2: (Color online) Transverse shapes of the nucleon, as in Fig. 1 except that $\phi_n = \pi$.

Another interesting possibility occurs in the Drell-Yan reaction $pp(\uparrow) \to l\bar{l}X$ using one transversely polarized proton [23]. In case the term h_{1T}^{\perp} causes a distinctive oscillatory dependence on the angle $3\phi - \phi_{S_1}$, where ϕ is the angle between the momentum of the outgoing lepton and the reaction plane in the lepton center of mass frame, and ϕ_{S_1} denotes the direction of polarization with respect to the reaction plane. The term h_{1T}^{\perp} is multiplied by the anti-quark Boer-Mulders function.

To illustrate the shapes that could be obtainable using this method we use the spectator model of [16] to evaluate the shapes of the proton. We rewrite Eq. (18) as

$$\frac{\sqrt{2}\rho_{\text{REL}T}(\mathbf{K}_T, \mathbf{n})}{\tilde{f}_1(K_T^2)} = 1 + \frac{\tilde{h}_1(K_T^2)}{\tilde{f}_1(K_T^2)}\cos\phi_n + \frac{1}{2}\frac{K_T^2}{M^2}\cos(2\phi - \phi_n)\frac{\tilde{h}_{1T}^{\perp}(K_T^2)}{\tilde{f}_1(K_T^2)},\tag{19}$$

where ϕ is the angle between \mathbf{K}_T and \mathbf{S}_T and ϕ_n is the angle between \mathbf{n} and \mathbf{S}_T . The transverse shapes of the nucleon, as defined by the right hand side of Eq. (19) are shown in Fig. 1, taking $\phi_n = 0$. Deformation is seen for values of K_T as small as 0.25 GeV, and this increases as K_T increases. Choosing $\phi_n = \pi$ emphasizes the non-spherical nature because the first two terms of Eq. (19) tend to cancel. The possible shapes implied by Eq. (19) can be thought of as transverse projections of the shapes displayed in Refs. [9]. One complication as that it would be very difficult to measure the necessary TMD's at all values of x to construct the integrals appearing here. However, the model [16] indicates that the

functions f_1, h_1 and h_{1T}^{\perp} have very similar x dependence, so that measurements at values of x for which these functions peak should be sufficient.

We have shown that that the non-spherical nature of the nucleon shape is closely related to the non-vanishing of the measurable TMD h_{1T}^{\perp} . While determining this function experimentally represents a challenge, the ultimate determination of a non-zero values would clearly demonstrate that the shape of the proton not round.

Acknowledgments

We thank the USDOE for partial support of this work, D. Boer, W. Detmold and L. Gamberg for useful discussions and J.C. Peng for a stimulating oral presentation.

- See the reviews: M. Anselmino, A. Efremov and E. Leader, Phys. Rept. 261, 1 (1995)
 [Erratum-ibid. 281, 399 (1997)]; B. Lampe and E. Reya, Phys. Rept. 332, 1 (2000)
 E. W. Hughes and R. Voss, Ann. Rev. Nucl. Part. Sci. 49, 303 (1999).
- [2] A. Airapetian et al. [HERMES Collaboration], arXiv:hep-ex/0609039; V. Y. Alexakhin et al.
 [COMPASS Collaboration], Phys. Lett. B 647, 8 (2007)
- [3] M. K. Jones et al. [Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. 84, 1398 (2000)
- [4] O. Gayouet al. Phys. Rev. Lett. 88, 092301 (2002)
- [5] P. Jain, B. Pire and J. P. Ralston, Phys. Rept. **271**, 67 (1996);
- [6] V. M. Braun, A. Lenz, N. Mahnke and E. Stein, Phys. Rev. D 65, 074011 (2002).
- [7] G. A. Miller and M. R. Frank, Phys. Rev. C 65, 065205 (2002)
- [8] A. V. Belitsky, X-d. Ji and F. Yuan, Phys. Rev. Lett. 91, 092003 (2003)
- [9] G. A. Miller, Phys. Rev. C 68, 022201 (2003), A. Kvinikhidze and G. A. Miller, Phys. Rev. C 73, 065203 (2006).
- [10] M.R. Frank, B.K. Jennings and G.A. Miller, Phys. Rev. C 54, 920 (1996).
- [11] K. Prokeš et al. Phys. Rev. B65, 144429 (2002)
- [12] P. J. Mulders and R. D. Tangerman, Nucl. Phys. B 461, 197 (1996) [Erratum-ibid. B 484, 538 (1997)].
- [13] J. Levelt and P.J. Mulders, Phys. Rev. **D49** (1994) 96
- [14] J.C. Collins and D.E. Soper, Nucl. Phys. **B194** (1982) 445
- [15] D. Boer and P. J. Mulders, Phys. Rev. D 57, 5780 (1998)
- [16] R. Jakob, P. J. Mulders and J. Rodrigues, Nucl. Phys. A 626, 937 (1997).
- [17] K. Goeke, A. Metz and M. Schlegel, Phys. Lett. B 618, 90 (2005).
- [18] M. Boglione and P. J. Mulders, Phys. Rev. D **60**, 054007 (1999).
- [19] V. Barone, A. Drago and P. G. Ratcliffe, Phys. Rept. 359, 1 (2002).
- [20] D. Boer, R. Jakob and P. J. Mulders, Nucl. Phys. B **564**, 471 (2000).
- [21] A. Bacchetta and P. J. Mulders, Phys. Rev. D 62, 114004 (2000).
- [22] M. Radici, R. Jakob and A. Bianconi, Phys. Rev. D 65, 074031 (2002).
- [23] D. Boer, Phys. Rev. D **60**, 014012 (1999)